Proceedings of 7th Windsor Conference: ***The changing context of comfort in an unpredictable world*** Cumberland Lodge, Windsor, UK, 12-15 April 2012. London: Network for Comfort and Energy Use in Buildings, http://nceub.org.uk

Numerical evaluation of radiative heat exchanges between human beings and cooling radiant systems

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Abstract: This paper presents a method of evaluating the radiative heat exchanges between the human body and surrounding surfaces. With the purpose to predict the detailed view factors for realistic human body shapes and complex configurations of the enclosures, an algorithm that calculates the view factors has been developed based on the Stokes' theorem.

The performance of the algorithm and its accuracy on predicting view factors for complex geometries are analyzed against analytical solutions or results obtained by other authors, using different methodologies. The algorithm is used for the evaluation of radiative heat exchanges between a sitting human being and a system of cooling radiant panels, for several positions and operating conditions of the radiative system.

The results obtained clearly demonstrate a high importance of the radiation heat transfer mechanism on the human thermal comfort. Significant effects of the position and the operating conditions of the radiative system are also observed.

Keywords: Thermal radiation; Radiative view factors; Stokes' theorem; Human / indoor environment interaction

Introduction

The use of radiant panels for achieving thermal comfort is a technology under expansion and it is expected that, in a near future, this systems will be the most common used in houses, office buildings and even in industry (Bean et al. 2010).

The radiative systems can be hydronic or electric. The hydronic solution is based on the installation of a network of tubes where hot or cold water circulates in a closed circuit. The heat is transferred as a result of the difference of temperatures. According to some authors (Watson and Chapman 2002; Woodson 2009) and manufactures ("Uponor" nd, "Zehnder Group UK - Homepage" nd), a radiant panel system is more efficient than a conventional system. In addition, the sensible loads are removed directly, promoting lower air velocities, reducing the sensation of draft and promoting a higher level of comfort.

In the specific case of a radiant panel system, the amount of heat transferred by radiation depends on several factors such as temperature of the emitting and receiving surfaces, emissivity, reflectance, transmittance and absorptance, and of the view factor between the panels and surfaces. Several studies have been done in order to quantify these radiative heat exchanges (Kakuta et al. 2001; Miyanaga 2001; A M Raimundo, Gaspar, et al. 2004; A. M. Raimundo, Gaspar, et al. 2004). In general, these authors aim to understand and quantify the human body response to different solutions of radiative systems, both in terms of location and operational conditions.

One of the possible keys for the evaluation of heat transfer by radiation between the human body and the environment is the determination of the view factors between: the surfaces involved, the person in a given body posture and the surrounding surfaces in an enclosure. However, the geometry of the human body is not so simple and the calculation of view factors with the environment is quite difficult.

In specific references it is possible to find a large number of expressions and / or graphics to obtain the view factors for simple geometric configurations. Several studies were developed in order to estimate the view factors, and thus the radiative heat transfer between human body segments or all body and the surroundings (Erchiqui and Ngoma 2007; Fanger et al. 1970; Ghaddar et al. 2006; A. M. Raimundo, Gaspar, et al. 2004). The most common methodologies used are the photographic methods, the integral area procedure, the *Gaussian* integration and the *Stokes*' theorem technique.

Raimundo et al. 2004 developed a computational code for the calculation of radiative exchanges where the calculation of view factors is done using numerical area integration, but restricted to Cartesian geometries and parallelepipedic shapes. The work presented here consists of the improvement of this pre-processor module. In order to improve and develop a more versatile radiation module, the Stokes' theorem is now applied in the calculation of the view factors. This new formulation allows the calculations for any room configuration and for more realistic human body physical models.

After the evaluation of the view factors and the calculations of the absorption factors on a pre-processor, the results obtained are used in a 3D-CFD simulation program, where the detailed convective and radiative heat exchanges are calculated. In this specific work it is studied the heat exchanges between the human body and the surrounding areas, provided by radiant cooling systems with two alternative locations: the ceiling and the walls.

Method

The key for the evaluation of heat transfer by radiation between the human body and the environment is the adequate calculation of view factors between the domain surfaces. The view factor, that represents the geometric relationship between two surface areas, can be defined by the generic definition. Considering two surfaces *i* and *j*, the view factor, F_{ij}, is described as the radiative fraction of energy that leaves the surface *i* and directly reaches the surface *j*. Thus, the view factor, F_{ij}, between a pair of surfaces is defined as (Sparrow and Cess 1978) :

$$F_{i\,j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi\,\delta^2} \, dA_i \, dA_j \tag{1}$$

where A_i and A_j are the areas of the surfaces *i* and *j*, respectively; δ is the length of the line that connects the elementary areas dA_i and dA_j on each surface; and θ_i and θ_j are the angles measured from the surfaces' normal to the line δ (**Error! Reference source not found.**).

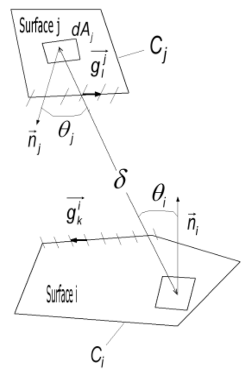


Figure 1. *View factor associated with radiation exchange between surfaces i and j.*

An alternative to equation (1) is the application of Stokes' theorem to the definition of view factor. Sparrow (1963) reduced the double area integral of view factor formula to the contour double integral formula (CDIF) using the Stokes' theorem. With this development, when the contours of both surfaces C_i and C_j are divided into the segment vectors g_k^i (k = 1, 2, …, nk) and g_l^j (l = 1, 2, …, nl), respectively, the view factor can be approximated by the following discretization expression (Kakuta et al. 2001):

$$F_{i\,j} = \frac{1}{2\pi A_i} \sum_{k=1}^{nk} \sum_{l=1}^{nl} \ln(\delta_{kl})\, g_k^i \, g_l^j \tag{2}$$

where δ_{kl} represents the distance between points on the respective contours. This methodology was found to be advantageous in terms of accuracy and computing time by several authors (Ambirajan and Venkateshan 1993; Erchiqui and Ngoma 2007; Kakuta et al. 2001; Shapiro 1985; Sparrow and Cess 1978).

Originally, the numerical evaluation of the contour integral in equation (2) consists of dividing the contours of the two surfaces into a finite number of line elements (see Figure 1). However, this formulation it is not sufficiently detailed to be applied in the evaluation of general indoor domains, with the presence of furniture, human beings, objects and rooms with non-regular polygonal shapes. In fact, for the detailed thermal analyze of such generic environments, it is necessary to take into account the total or partial radiant obstructions created by obstacles located between two particular surfaces. For example, with the formulation of equation (2), it is quite difficult to implement an algorithm that adequately considers the effect of obstacles that induce a partial blinding effect between two surfaces.

In order to overcome this important limitation, in this work is used an original formulation, also based on the *Stokes*' theorem but applied to elementary areas. This implies the division of the surfaces into a grid of elementary areas as represented in Figure 2. Then, instead of an overall determination of the view factor between two global surfaces, the *Stokes*' theorem is applied to the determination of view factors between elementary areas.

After testing several possible formulations for the numerical evaluation of elementary areas view factors (F_{kl}), the best results were obtained using the following expression:

$$F_{kl} = \frac{1}{2\pi A_i} \left| \sum_{k=1}^{4} \sum_{l=1}^{4} \ln(\delta_{kl})\ g_k^i\ g_l^j \right| \tag{3}$$

where δ_{kl} represents the distance between the middle points on the respective side of elementary areas contour. The summing's operator ranged from 1 to 4 because the elementary areas used are assumed quadrilateral. If a particular elementary area has a triangular shape it is assumed "mathematically" quadrilateral considering one side with no length. The imposition of a positive result is necessary because the scalar product can outcome negative values for the view factors between elementary areas. Basing the calculation on a grid of the elementary areas, the algorithm can verify if there is any obstacle between a pair of elementary areas and take into account its influence.

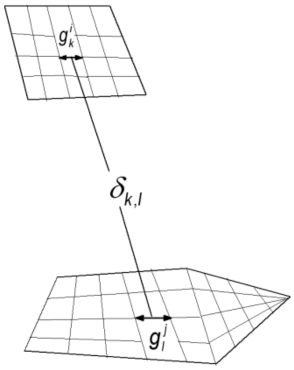


Figure 2. *Schematic representation of the application of Stokes' theorem to the elementary surfaces.*

Considering the additive nature of the view factors, then the view factor F_{ij} from an arbitrary original global surface *i* (divided into l_i elementary areas) to an arbitrary surface *j* (divided into k_j elementary areas) is calculated using the following four steps:

1st. Calculation of the view factors from all the elementary areas *k* to the original surface *j*: $F_{kj} = \sum_{l=1}^{nj} F_{kl}$.

2nd. Using the reciprocity relationship: $F_{jk} = \frac{A_k}{A_j} F_{kj}$.

3rd. Calculation of view factors from the original surface j to the original surface i: $F_{ji} = \sum_{k=1}^{ni} F_{jk}$.

4th. Now, using the reciprocity relationship: $F_{ij} = \frac{A_j}{A_i} F_{ji}$

After the calculation of the view factors, the Gebhart absorption factors (G_{ij}) are calculated making use of these results. The absorption factor between two different surfaces (i,j) is defined as the fraction of the radiation flux emitted by the surface i that is absorbed by the surface j, coming to this one from every possible paths, including successive reflections (Gebhart 1961).

Considering that the domain has N surfaces and that all surfaces are grey and perfectly diffuse ($\varepsilon = \alpha$) with known emissivity, the Gebhart absorption factors can be obtained by the resolution of a linear N^2 system of equations with equal number of unknowns, of the type:

$$G_{ij} = F_{ij}\varepsilon_j + \sum_{k=1}^{N} \left(F_{ik}\rho_k G_{kj}\right) \tag{4}$$

where i = 1 to N and j = 1 to N.

Within the 3D-CFD program a calculation module makes use of the Gebhart absorption factors to obtain the radiative exchanges. This inner module interacts internally with the CFD code and, on each iteration of the global algorithm, is responsible for the determination of radiative heat exchanges between the different surfaces. Starting with the Gebhart absorption factors, previously calculated by the pre-processor and the surface temperatures estimated within each iteration, this module ensures the determination of the net radiant heat flux transferred from surface i to surface j:

$$\dot{Q}_{ij} = A_i \varepsilon_i \sigma G_{ij}\left(T_i^4 - T_j^4\right) \tag{5}$$

Then, the net radiant heat flux lost from the surface i can be obtained from:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{ij} \tag{6}$$

Testing and validation

In order to validate and evaluate the performance of the algorithm, some comparisons were done with analytical solutions and with results obtained by other authors. For example, Erchiqui and Ngoma (2007) have tested four different methods - double integral area, quadratic *Gauss* integration, contour integral and a semi-analytical method (developed by them). The following two configurations were considered:

- Two parallel surfaces, dimensions 1 × 1 m^2 and at 1 m of distance;
- Two perpendicular surfaces, dimensions 1 × 1 m^2, with a common edge.

According to their methods, for the double integral area the surfaces were divided into 16 elementary areas, for the quadratic *Gauss* integration a maximum of seven points was admitted while for the integral contour and the semi-analytical methods a maximum of five points was considered.

The results obtained with these methods were compared with analytical solutions. Table 1 shows the results of (Erchiqui and Ngoma 2007), those obtained by the present formulation and the analytical solution.

Table 1: *Comparison of view factors for parallel and perpendicular surfaces obtained using the present methodology and those proposed by Erchiqui and Ngoma (2007) - first four lines - and the analytical formulation (last line).*

	Calculation method	Number of subdivisions	Parallel surfaces		Perpendicular Surfaces	
			Results	**Error (%)**	**Results**	**Error (%)**
Erchiqui and Ngoma (2007)	**Double integral area**	16 (Elem areas)	0.204724	2.45	0.22883	14.39
Erchiqui and Ngoma (2007)	**Quadratic Gauss**	7 (Pts Gauss)	0.20013	0.15	0.479857	139.88
Erchiqui and Ngoma (2007)	**Contour Integral**	5 (Pts Gauss)	0.199835	0.014	Diverge	Diverge
Erchiqui and Ngoma (2007)	**Semi-analytical**	3 (Pts Gauss)	0.199829	0.00	0.200041	0.00
	Present formulation	16 (Elem areas)	0.195727	2.09	0.200792	0.37
	Analytical solution	-	0.199825	-	0.200044	-

From Table 1 it is possible to verify that the semi-analytical formulation developed by Erchiqui and Ngoma (2007) is the one with lower relative errors. The contour integral method presents low relative errors for parallel surfaces but, due to logarithmic singularity, diverges for the situation with a common edge. The double integral area and the quadratic *Gauss* integration present the higher values. With the formulation presented in this paper the relative error is about 2.1% in the case of parallel surfaces and 0.37% in the case of a common edge. The present formulation, that makes use of the contour integral as stated before, leads to better results when compared with the double integral area.

Case study

The configuration under analysis in the present work comprises a typical office room ($2.86 \times 4.70 \times 2.96$ m^3) with a person, a desk, a chair and a computer (Figure 3). Under the desk there is a drawers unit. The door is located on the right wall and the occupant is seated about 70 cm away from the back wall.

The occupant faces towards the north wall. In the east wall (right of the individual) there is a door and on the west a window. The west wall and the ceiling contact directly with the outside. The east wall separates the room from a corridor. The north and south walls and the floor are in contact with other offices at the same thermal conditions. It is assumed the existence of two cracks that are responsible for the ventilation of the space, one horizontal under the door and other vertical in the middle of the window.

The particular situations considered in the present work deal with the geometrical conditions illustrated in Figure 4. The radiant cooling panel can be placed on the ceiling or on the front or back walls. When placed on the ceiling, the panel could span an area of 40 or 80% of ceiling surface. In the case of the walls, the height of the panel is limited to 1.5 m high and can occupy 20 or 40% of the wall surface (Figure 4).

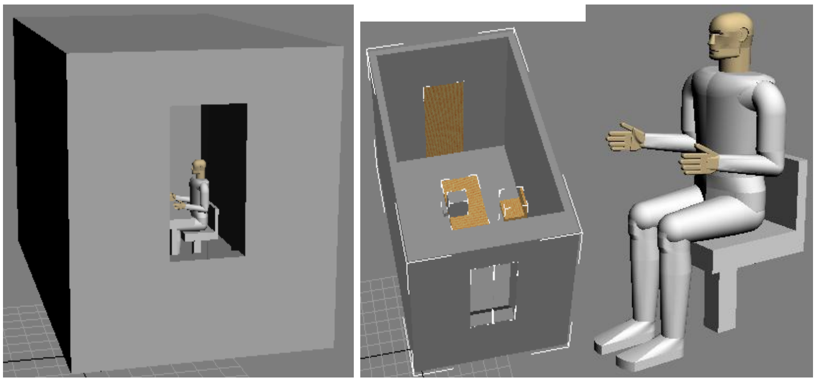
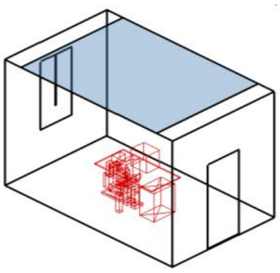
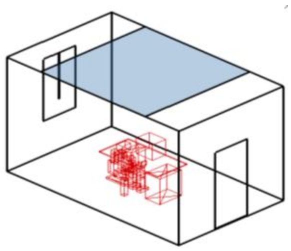


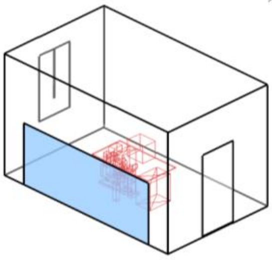
Figure 3. *Case study without the cooling radiant system.*



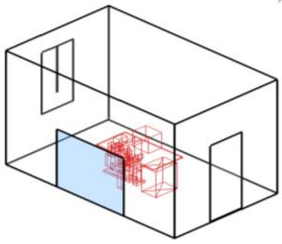
(*a*) 80% of the ceiling (cases 2 and 3).



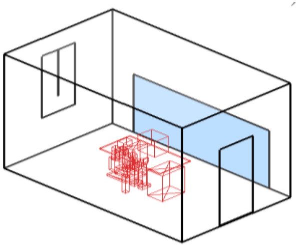
(*b*) 40% of the ceiling (cases 4 and 5).



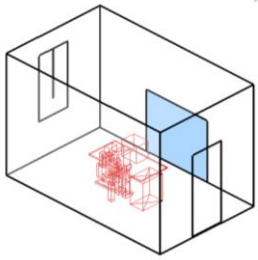
(*c*) 40% of the south wall (cases 6 and 7).



(*d*) 20% of the south wall (cases 8 and 9).



(*e*) 40% of the north wall (cases 10 and 11).



(*f*) 20% of the north wall (cases 12 and 13).

Figure 4. *Schematic representation of the cases under study.*
Case 1 - without radiant panels (reference situation).

Results and Discussion

In Table 2 is shown the view factors from the complete human body and the remaining surfaces of the domain (obtained using the new pre-processor).

As it can be seen, the highest view factor is between the human body and itself (0.26), with or without the cooling system. It deserves to be highlighted the value of 0.19 for the view factor from the whole body to the floor and the value of 0.14 to the south wall (on the back of the person).

Table 2. *View factors from the human body to the remaining surfaces.*

	Without panels	Ceiling panel area		South panel area		North panel area	
		80%	**40%**	**40%**	**20%**	**40%**	**20%**
Human body	0.26	0.26	0.26	0.26	0.26	0.26	0.26
Left wall (with window)	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Right wall (with door)	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Front wall	0.10	0.10	0.10	0.10	0.10	0.05	0.06
Back wall	0.14	0.14	0.14	0.04	0.07	0.14	0.14
Ceiling	0.08	0.01	0.02	0.08	0.08	0.08	0.08
Floor	0.19	0.19	0.19	0.19	0.19	0.19	0.19
Radiant panels	-	0.07	0.06	0.10	0.07	0.05	0.04
Table	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Drawers unit	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Chair	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Display	0.01	0.01	0.01	0.01	0.01	0.01	0.01
CPU	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Without radiant system the higher view factors are between the human body and the floor, followed by the view factors between the back and front walls, respectively. The chair, due to the contact of the human body, is the piece of furniture with higher view factor.

When the radiant panel is considered, the view factor between the human body and the panel is higher when it is located on the closest wall, followed by the situation where the panel system is mounted on the ceiling. The solution with the panel located in front of the human body corresponds to the lowest view factors between the human body and the radiant system. As expected, in terms of radiative heat exchanges between the human body and the panel system it was predicted that, for the same thermal conditions, they are proportional to the corresponding view factors.

The radiative heat flux losses by the whole human body, in a hot summer situation and for the thirteen situations tested, are presented in Table 3.

Table 3. *Whole human body radiation heat losses.*

Case	Panel localization	Occupied area [%]	Panel temperature [ºC]	Radiation losses [W]
1	-	-	-	39.70
2	Ceiling	80%	22	51.58
3			19	57.71
4		40%	22	48.36
5			19	52.34
6	South wall (on the back of occupant)	40%	22	49.93
7			19	61.17
8		20%	22	45.19
9			19	49.30
10	North wall (in front of occupant)	40%	22	48.08
11			19	55.53
12		20%	22	42.89
13			19	48.43

Case 1 – Without cooling system

The results of the air temperature field around the human body, obtained using the 3D-CFD code, for the situation without the cooling radiant system is shown in Figure 5.

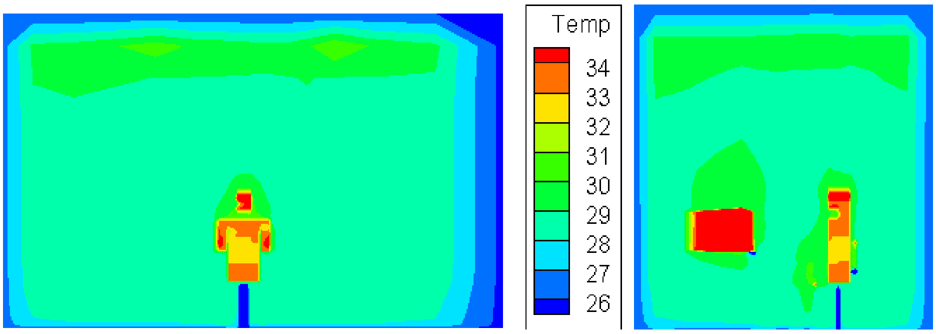


Figure 5. *Air temperature distribution around the human body (situation without radiant cooling system).*

It can be seen that the air temperature in the room is somehow uniform, except near the walls (especially the east one) and within the obstacles/heat sources (lights, human body and computer).

The sensible heat losses of each section of the human body to the environment are present in Figure 6 for the situation without radiant system. The non-symmetry of the results is related with the asymmetry of the air flow in the room, as well as the temperatures of the surrounding surfaces.

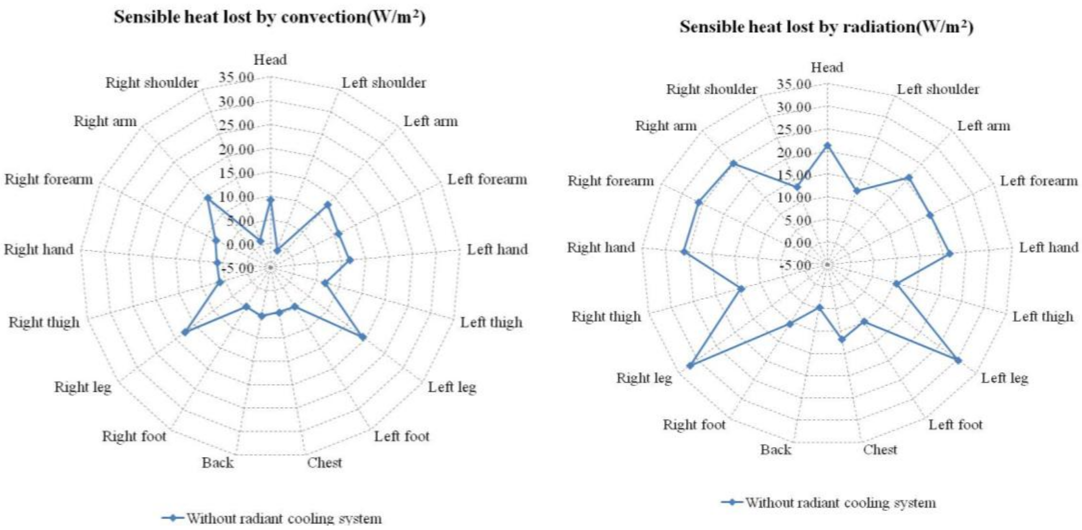


Figure 6. *Sensible heat losses (situation without radiant cooling system).*

In terms of the convective heat flux, the highest values were found in the legs, the head, the arms, the forearms and the left hand. The left side of the human body has higher values when compared to the right one. The highest value occurs in the left leg with about 19 W/m^2. The lowest values occur in the back and the chest, resulting from the thermal insulation of clothing and the protection provided by the chair.

The radiative heat exchange is higher than the convective one, particularly for the right part of the body (closest to the east wall). Remember that the view factors with the east wall are also higher than with the west one (see Table 2).

Making an overall assessment, the situation without cooling system provides a total sensible human body heat loss of 59.63 W, where 19.93 W correspond to convective exchanges and 39.70 W to radiative ones. Assuming an activity of 1.2 met and a clothing insulation of 0.63 clo results a comfort predicted mean vote of *PMV* = 1.94, suggesting an uncomfortable hot situation.

Case 3 –Panel at 19ºC occupying 80% of the ceiling area

From the range of the 13 configurations tested, the best result in terms of human thermal comfort was obtained for Case 3, corresponding to the situation of a cooling radiant system with panels at 19ºC and occupying 80% of the ceiling area. The corresponding air temperature distributions in plane sections passing through the body are shown in Figure 7. Now, the average air room temperature is about 0.9ºC lower than the situation without cooling system (reference case).

For this particular situation, the application of a radiant cooling system allows a greater stratification of room air temperatures, mainly at the floor level and near to the east. Also, the air temperatures around the human body are lower compared with the reference situation.

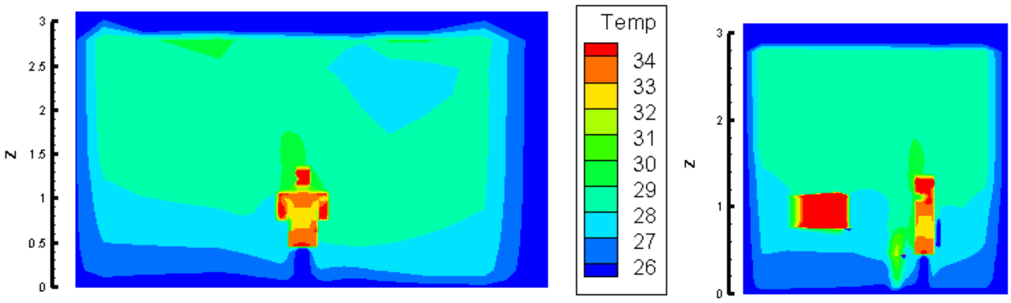


Figure 7. *Air temperature distribution around the human body for Case 3 (panel at 19ºC occupying 80% of ceiling area).*

A comparison of the convective and the radiative heat losses of each human body segment between Case 1 (without radiant system) and Case 3 (panels at 19ºC and occupying 80% of the ceiling area) is shown in Figure 8.

As expected, in relation to the reference situation, the sensible heat losses of the human body increases due to the operation of a radiant cooling system located on the ceiling. Contrary to what was expected, the radiative exchange did not increase significantly. On the other hand, the convective heat fluxes increase considerably in the limbs, especially the legs.

Making again an overall assessment, the situation with the radiant cooling panels at 19ºC occupying 80% of the ceiling provides a total sensible human body heat loss of 132.76 W, where 75.05 W correspond to convective exchanges and 57.71 W to radiative ones. For an activity of 1.2 met and a clothing insulation of 0.63 clo results *PMV* = -0.11, suggesting a very comfortable situation.

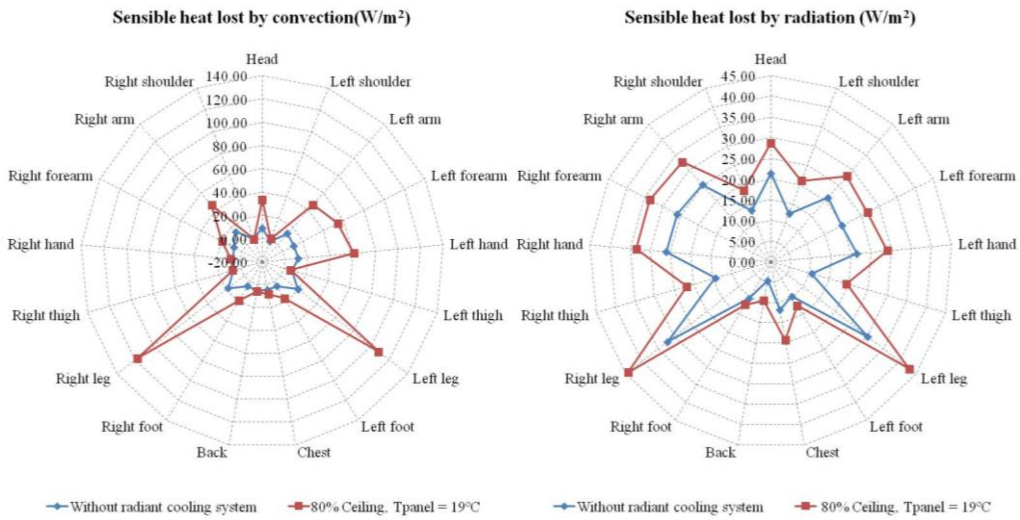


Figure 8. Human body segments sensible heat losses:
- Blue line – without radiant system (Case 1),
- Brown line – with radiant panels at 19ºC occupying 80% of the ceiling (Case 3).

Conclusions

The main objective of this work is focused on the evaluation of heat exchanges between the human body and the surrounding surfaces provided by radiant cooling systems. It also intends to evaluate the response of the human body when the same radiative system is placed at different locations and for diverse operating conditions.

An algorithm that makes use of Stokes' theorem has been developed and implement in a pre-processor for a 3D-CFD code. The performance of the algorithm and its accuracy on predicting view factors was analyzed against analytical solutions and results obtained by other authors using different methodologies. The good agreement achieved, namely with the analytical solutions, suggest that the present algorithm has very good predicting capabilities and accuracy.

The methodology developed was applied to calculate de view factors between the human body and the surroundings in a radiant cooled environment. Thirteen different cases were studied for a typical summer hot situation. One, considered as reference, with no kind of cooling system. Twelve with an active cooling radiant system, running with two alternative temperatures (22 or 19ºC), located at different places in the room and having different areas.

It can be concluded that the use of radiant cooling systems helps the improvement of the human body thermal comfort. The results clearly showed that the best solution is the use of a radiant system with panels at 19ºC occupying 80% of the ceiling area.

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